The New Screening Characteristics of Strongly Non-ideal and Dusty Plasmas. Part 3: Properties and Applications

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Abstract

The physical sense and the properties of a group screening parameters which are determined in the previous papers (Part 1 and Part 2) for single- and two-component systems is discussed in this paper. On the base of data from the mentioned papers are determined two new characteristic lengths which complete a new hierarchy system of screening lengths in plasma. It was shown that the methods developed in Part 1 and Part 2 generates results which are applicable to the strongly non-ideal gaseous and dusty plasmas, and manifest a very good agreement with the existing experimental data.

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I. INTRODUCTION

In the previous papers [1, 2], here Part 1 and Part 2, are developed the methods for describing of the electrostatic screening in single- and two-component systems (gas of electrons on a positive charged background, some electron-ion and dusty plasmas etc.). These methods generate a group of the new screening parameters which characterize the considered systems. In this paper the physical sense of these parameters is discussed, and the reinterpretation of some known screening parameters (Landau radius, non-ideality parameters etc.) is given. On the base of results from Part 1 and Part 2 in this paper are obtained two new screening parameters which provide the possibility to form a new hierarchy system of the characteristic screening lengths and to compare the obtained results with the existing experimental data.

The material presented in this paper is distributed in four Sections and one Appendix. In Section II are given the some quantities and relations from Part 1 and Part 2 which should make easier reading of this paper; in Sections III and IV are considered "small" characteristic lengths and connected with them non-ideality parameters for single- and two-component systems. Besides, in the same Sections are introduced "medium" and "large" characteristic lengths for the considered systems. The obtained results are compared with the existing experimental data in Section IV. The conclusions of this paper are given in Section V. Finally, one important example of the application of obtained results, related to the systems with more than two components, is considered in Appendix A.

II. REMARKS: THE MAIN QUANTITIES AND RELATIONS

A. Single-component systems

The density, temperature and the charge of free particles in the initial (homogeneous) system: N, T and Ze, where $Z = \pm 1, \pm 2...$, and e - the modulus of the electron charge;

The screening constant:

$$\kappa \equiv \frac{1}{r_\kappa} = \left[\frac{4\pi (Ze)^2}{\partial\mu/\partial N}\right]^{\frac{1}{2}}, \tag{2.1}$$

where $\partial\mu/\partial N \equiv \partial\mu(N,T)/\partial N$, $\mu(N,T)$ is the chemical potential, and r_κ- the corresponding characteristic length;

The charge density in the accessory (non-homogeneous) system with the probe particle:

$$\rho(r) = \rho_b + Ze \cdot n(r), \tag{2.2}$$

where $\rho_b = -Ze \cdot N$ is the background charge density, and $n(r)$ - the free particle density at the distance r from the origin of the chosen reference frame;

In the region $r \geq r_0$ we have that

$$\rho(r) = -ZeN \cdot r_0 \cdot \exp(\kappa r_0) \cdot \frac{\exp(-\kappa r)}{r}. \tag{2.3}$$

The radius r_0 and the parameters $\gamma_s(x)$ and $\gamma_\kappa(x)$:

$$r_0 = r_s \cdot \gamma_s(x), \qquad r_0 = r_\kappa \gamma_\kappa(x), \tag{2.4}$$

$$\gamma_s(x) = [(1 + x^3)^{\frac{1}{3}} - 1]/x, \qquad \gamma_\kappa(x) = (1 + x^3)^{\frac{1}{3}} - 1, \tag{2.5}$$

$$x = \kappa r_s = \frac{r_s}{r_\kappa}, \qquad r_s = \left(\frac{3}{4\pi N}\right)^{\frac{1}{3}} \tag{2.6}$$

where r_s is Wigner-Seitz's radius (the radius of the sphere with the volume $1/N$);

The condition of electro-neutrality of the accessory system:

$$Ze + \int_0^\infty \rho(r) \cdot 4\pi r^2 dr = 0, \tag{2.7}$$

where Ze is the charge of the fixed probe particle.

B. Two-component systems

The density, temperature and the charge of ions in the initial (homogeneous) system: N_i, T_i and $Z_i e$, where $Z_i = 1, 2...$;

The density, temperature and the charge of electrons in the same system: N_e, T_e and $-e$ or $Z_e e$, where $Z_e = -1$;

The ion and electron screening constants and the characteristic lengths:

$$\kappa_{i,e} \equiv \frac{1}{\kappa_{i,e}} = \kappa_{0;i,e} \cdot (1 - \alpha)^{\frac{1}{2}}, \qquad \kappa_{0;i,e} = \left[\frac{4\pi (Z_{i,e} e)^2}{\partial \mu_{i,e} / \partial N_{i,e}}\right]^{\frac{1}{2}}, \tag{2.8}$$

where $\partial\mu_{i,e}/\partial N_{i,e} \equiv \partial\mu_{i,e}(N_{i,e}, T_{i,e})/\partial N_{i,e}$, $\mu_i(N_i, T_i)$ and $\mu_e(N_e, T_e)$ are the ion and electron chemical potentials, α is the electron-ion correlation coefficient given by expression

$$\alpha = 1 - \frac{\frac{2}{3}x_s^3}{(1 + x_s)\exp(-x_s) - (1 - x_s)\exp(x_s)}, \tag{2.9}$$

and $x_s = \kappa_{0;e} r_{s;i}$;

The charge densities in the accessory (non-homogeneous) systems with the probe particles:

$$\rho^{(i,e)}(r) = Z_i e \cdot n_i^{(i,e)}(r) - e \cdot n_e^{(i,e)}(r), \tag{2.10}$$

where $n_i^{(i,e)}(r)$ and $n_e^{(i,e)}(r)$ are the ion and electron densities at the distance r from the origin of the chosen reference frame; upper indexes (i, e) pointed the considered case: (i) - the probe particle charge is equal to $Z_i e$, (e) - the probe particle charge is equal to $-e$;

In the region $r \geq r_{s;i,e}$ we have that

$$\rho^{(i,e)}(r) = Z_{e,i} e(1 - \alpha) \cdot N_{i,e} r_{0;i,e} \cdot \exp(\kappa_{i,e} r_{0;i,e}) \cdot \frac{\exp(-\kappa_{i,e} r)}{r}, \tag{2.11}$$

where $Z_e = -1$.

The radii $r_{0;i,e}$ in the cases (i) and (e) and the parameters $\gamma_s(x_{i,e})$ and $\gamma_\kappa(x_{i,e})$:

$$r_{0;i,e} = \gamma_s(x_{i,e}) \cdot r_{s;i,e}, \qquad r_{0;i,e} = \gamma_\kappa(x_{i,e}) \cdot r_{\kappa;i,e} \tag{2.12}$$

$$x_{i,e} = \kappa_{i,e} r_{s;i,e} = \frac{r_{s;i,e}}{r_{\kappa;i,e}}, \qquad r_{s;i,e} = \left(\frac{3}{4\pi N_{i,e}}\right)^{\frac{1}{3}} \tag{2.13}$$

where $r_{s;i,e}$ are Wigner-Seitz's radii (the radii of the spheres with the volumes $1/N_i$ and $1/N_e$);

The electro-neutrality conditions of electro-neutrality of the accessory systems:

$$Z_{i,e} e + \int_0^\infty \rho^{(i,e)}(r) \cdot 4\pi r^2 dr = 0, \tag{2.14}$$

where $Z_{i,e} e$ is the charge of the corresponding probe particle.

III. THE SCREENING PARAMETERS OF THE SINGLE-COMPONENT SYSTEMS

A. "Small" characteristic length r_0 and the non-ideality parameters $\gamma_{s,k}$

The connection of r_0 with Landau radius r_L. As it was announced in Part 1,

all characteristic lengths of the developed method appear already in the case of single-component systems. The first of them is "small" characteristic length r_0, given by Eqs. (2.4)-(2.6), which is interpreted as a radius of the sphere centered on the probe particle and classically forbidden for other free particles. Because of that it was useful to compare the radius r_0 with a known characteristic length which has similar physical sense. Here, we keep in mind Landau's radius

$$r_L = \frac{(Ze)^2}{kT} \tag{3.1}$$

which is used in the case of the classical systems (see e.g. [3]). In accordance with Eqs. (2.4)-(2.6) we have that

$$r_0 = \frac{1}{3}\kappa^2 r_s^3 \cdot \left(1 + O(x^3)\right), \qquad x \ll 1, \tag{3.2}$$

where in the classical case ($\partial\mu/\partial N = kT/N$) the relation

$$\frac{1}{3}\kappa^2 r_s^3 = \frac{(Ze)^2}{kT} \tag{3.3}$$

is valid. So, in this case the radii r_0 and r_L are connected by relation

$$\lim_{x\to 0} \frac{r_0}{r_L} = 1. \tag{3.4}$$

It means that r_L represents an approximation od the characteristic length r_0 which is applicable in the region of small x. Consequently, *the parameter r_0 can be treated as the generalization of Landau's characteristic length r_L which introduces into consideration from physical reasons.* Let us emphasize that, contrary to r_L which is principally unlimited, the radius $r_0 < r_s$ for any $\kappa r_s > 0$, in accordance with the conditions in real physical systems require, and that Wigner-Seitz's radius $r_s = \lim_{x\to\infty} r_0$.

The connection of $\gamma_{s,k}$ with non-ideality parameters Γ and γ. From Eqs. (2.4)-(2.6), (3.2) and (3.4) follow the expressions

$$\gamma_s = \frac{r_0}{r_s}, \qquad \gamma_\kappa = \frac{r_0}{r_\kappa}, \tag{3.5}$$

for the coefficients γ_s and γ_k. In connection with these expressions in the classical case one should keep in mind the relation (3.4) and the fact that $r_\kappa = r_D$, where r_D is the corresponding Debye's radius. With respect to this it is useful to compare Eqs. (3.5) with the expressions for known quantities

$$\Gamma = \frac{r_L}{r_s} = \frac{(Ze)^2}{kT \cdot r_s}, \quad \gamma = \frac{r_L}{r_D} = \frac{(Ze)^2}{kT \cdot r_D}, \tag{3.6}$$

which usually use as the non-ideality parameters for the classical systems. Usually the parameters Γ and γ, similarly to Landau's radius r_L, introduce from some physical reasons (see for example [3, 4]). From Eqs. (2.4)-(2.6), (3.5) and (3.6) it follows that in the classical case

$$\lim_{x\to 0}\frac{\gamma_s}{\Gamma}=1, \qquad \lim_{x\to 0}\frac{\gamma_\kappa}{\gamma}=1, \tag{3.7}$$

where $x = \kappa r_s$. It means that the parameters Γ and γ appear here as approximative values of the coefficients γ_s and γ_κ in the classical case in the region $x \ll 1$. Consequently, *the parameters γ_s and γ_κ can be treated as the generalized non-ideality parameters of single-component systems with any N, T and Z which make possible their non-relativistic treatment.*

From Eqs. (2.5) and (2.6) it follows that the coefficients γ_s and γ_κ represent the functions of the parameter r_s/r_κ which is closely connected with another known classical quantity. Namely, in the classical case ($r_\kappa = r_D$) we have that $(r_s/r_D)^{-3} = n_D$, where n_D is so called Debye's number, i.e. the mean number of the free particles in the sphere with the radius r_D.

The behavior of the coefficients γ_s and γ_κ and the parameters Γ and γ as functions of κr_s is illustrated in Fig. 1. This figure shows that in the region $\kappa r_s \leq 0.5$ the values of parameters Γ and γ are practically same as the values of γ_s and γ_κ and. They values left very closed up to the value $\kappa r_s = 1$.

B. "Medium" characteristic length r_c and the radius r_κ

The charges $Q_{in,out}(r)$. The electro-neutrality condition Eq. (2.14) can be presented in the form: $Q_{in}(r) + Q_{out}(r) = 0$, where $0 \leq r < \infty$, and the quantities $Q_{in,out}(r)$ are given by expressions

$$Q_{in}(r) = Ze + \int_0^r \rho(r')4\pi r'^2 dr', \qquad Q_{out}(r) = \int_r^\infty \rho(r')4\pi r'^2 dr', \tag{3.8}$$

with the charge density $\rho(r)$ defined by Eq. (2.2). One can see that $Q_{in}(r)$ and $Q_{out}(r)$ represent the total charge inside the sphere with radius r, centered in the point O, and the total charge of the rest of the space, respectively.

Probably, the quantities $Q_{in,out}(r)$ could have different applications. So, the fact that in the considered case the total average charge of the probe particle self-sphere, i.e. $Q_{in}(r = r_s)$, is strictly equal to the average charge of the free particles inside this sphere, can be of interest.

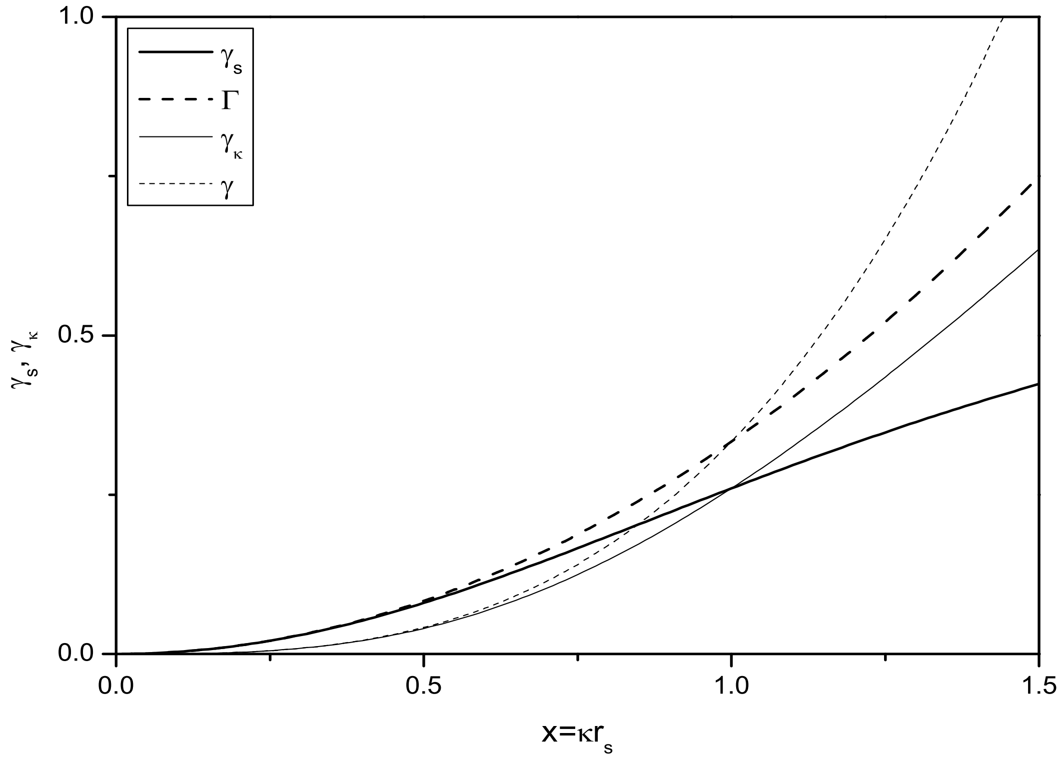

FIG. 1: The behavior of the parameters $\gamma_s = r_0/r_s$ and $\Gamma = r_L/r_s$, and $\gamma_\kappa = r_0/r_\kappa$ and $\gamma = r_L/r_D$, defined by Eqs. (2.4)-(2.6) and (3.6), as functions of x, in the classical case.

However, in the further consideration the quantity $Q_{in}(r)$ will play only an accessory role. Namely, it will be used only the fact that $Q_{in}(r)$, obtained by means Eqs. (2.2) and (3.8), has the physical sense in the whole space (including the region $r < r_s$), and can characterize the degree of the probe particle charge screening.

The connection of r_c with the radius r_κ. In the case of single-component systems the screening constant κ, defined by Eq. (2.1), represents one of the main parameters in both methods - the method developed in this work and Debye-Hückel's (DH) method. Consequently, the characteristic lengths $r_\kappa = 1/\kappa$ has to be treated in the similar way.

Within DH method it is usual to interpret r_κ, which is equal to Debye's radius r_D in the classical case, as the screening radius. That is based on the fact that r_κ is a distance from the probe particle where the DH electrostatic potential (described in Part 1) becomes less

than Coulomb potential of this particle by the factor e^{-1}.

Here we will consider the role of radius r_κ from other aspect in order to clarify its real physical sense in both methods and determine the region of its applicability. For this purpose, we will introduce the radial charge density $P(r) = 4\pi r^2 \cdot \rho(r)$, where $\rho(r)$ is given by Eq. (2.2), and the characteristic length r_c defined by relation

$$|P(r_c)| = \max_{0<r<\infty} |P(r)|. \tag{3.9}$$

The behavior of $P(r)$ for several values of $x = \kappa r_s$ is shown in Fig. 2. This figure illustrates the fact that for $\kappa r_s < 7^{\frac{1}{3}}$ the point $r = r_c$ there is in the region $r > r_0$, while for $\kappa r_s \geq 7^{\frac{1}{3}}$ we have that $r_c = r_0$. Since in the region $r > r_0$ the charge density $\rho(r) \sim \exp(-\kappa r)/r$, the parameter r_c for $\kappa r_s < 7^{\frac{1}{3}}$ represents the root of equation

$$\frac{dP(r)}{dr} = 0, \tag{3.10}$$

where $P(r) \sim r \cdot \exp(-\kappa r)$. Consequently, in this region the parameter r_c is equal to r_κ. It means that within the method developed in this work we have the relations

$$r_c = \begin{cases} r_\kappa, \;\; 0 < \kappa r_s < 7^{\frac{1}{3}}, \\ r_0, \;\; 7^{\frac{1}{3}} \leq \kappa r_s < \infty, \end{cases} \tag{3.11}$$

which determine the real physical sense of the radius r_κ. Namely, from Eq. (3.11) it follows that r_κ has the physical sense only in the region $\kappa r_s < 7^{\frac{1}{3}}$ where $r_\kappa = r_c$, while in the region $\kappa r_s > 7^{\frac{1}{3}}$ the length r_κ loses any connection with the charge distribution in the probe particle neighborhood and, consequently, loses any physical sense. *Let us emphasize that on the base of above mentioned one can conclude that it is always $r_c \geq r_0$.*

Then, we will consider the problem of screening of the probe particle as a problem of compensation of its charge Ze within the sphere of a radius r centered at the probe particle. For that purpose we will draw attention to the fact that uncompensated part of this charge in the case of such a sphere is equal to the charge $Q_{in}(r)$. Keeping this in mind, we compared the charge $Q_{in}(r_\kappa)$ with the charge Ze in the region $r_k > r_0$. Using the relations (3.8) and (2.2) it could be shown that in this whole region $Q_{in}(r_\kappa) > 0.735 \cdot Ze$. From here it follows that *the radius r_κ, as well as the radius r_c, cannot be treated as a characteristic length of full screening (neutrality) of the probe particle charge.* Because of that, such a characteristic length is determined here in another way.

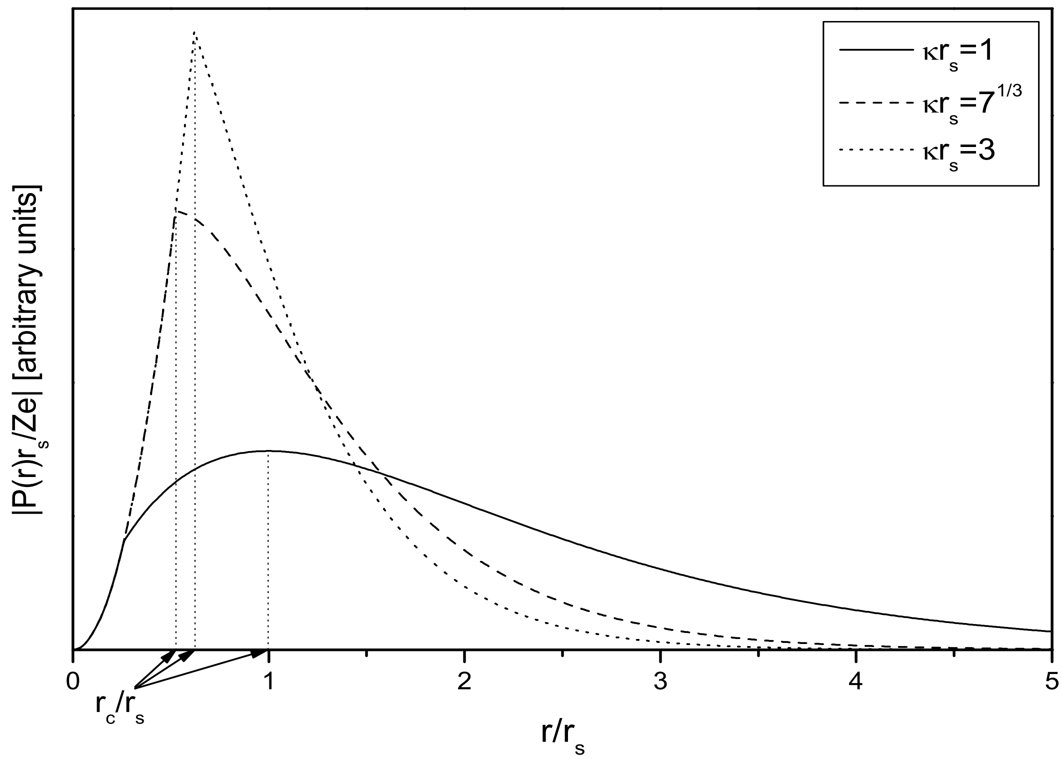

FIG. 2: The behavior of the radial charge density $P(r) = 4\pi r^2 \rho(r)$ for the cases: $\kappa r_s = 1$, when $\kappa r_0 < 1$; $\kappa r_s = 7^{\frac{1}{3}}$, when $\kappa r_0 = 1$; $\kappa r_s = 3$, when $\kappa r_0 > 1$. The points where $P(r)$ has the maximal value are denoted by r_c.

C. "Large" characteristic length r_n

The charge $Q_{in}(r)$ and the quantity $\nu(r)$. In accordance with Eq. (2.2) and the condition Eq. (2.7), the charge $Q_{in}(r)$ can be presented in the form

$$Q_{in}(r) = -Q_{out}(r) = Ze \cdot \nu(r), \tag{3.12}$$

where the quantity $\nu(r)$ is given by the expression

$$\nu(r) = \int_r^\infty [N - n(r')] \cdot 4\pi r'^2 dr', \tag{3.13}$$

and $n(r)$ is the free particle density in the considered system with the probe particle.

Although the expressions (3.13) for $\nu(r)$ can be determined for any $r > 0$, this quantity has a special physical meaning in the region $r \geq r_s$. Namely, it can be shown that in this region $\nu(r)$ represents the mean number of particles whose surplus into the mentioned sphere of radius r and corresponding deficit in the rest of the space causes the deviations of $Q_{in}(r)$ and $Q_{out}(r)$ from zero. Because of that only the quantities $Q_{in}(r)$ and $\nu(r)$ in the region $r_s \leq r \leq \infty$ will be needed in this paper. From this reason in further considerations we will use the expression for $\nu(r)$ which is applicable in the region $r_0 < r < \infty$, since it is always $r_0 < r_s$. In accordance with Eq. (20) for $n(r)$ from Part 1 we have that

$$\nu(r) = (1 + \kappa r) \exp(-\kappa r) \cdot \chi(x), \qquad r > r_0, \tag{3.14}$$

where $x = \kappa r_s$, and $\chi(x)$ is defined by Eq (30) and illustrated by Fig. 3 from Part 1.

Similarly to $Q_{in}(r)$, the quantity $\nu(r)$ could have also some different applications. For example, knowing of $\nu(r = r_s)$ makes possible to estimate the density N_p of pairs of particles with the inter-particle distance less then r_s. Namely, in the binary approximation (not more than one particle inside the self-sphere of any particle in the initial system) it can be shown that: $N_p \cong (1/2) \cdot N \cdot \nu(r_s)$, where $\nu(r_s)$ is given by Eq. (3.14) with $r = r_s$. However, in further consideration the charge $Q_{in}(r)$ and the quantity $\nu(r)$ will play only an accessory role.

The characteristic length r_n as the neutrality radius. In order to determine the required "large" characteristic length one should consider the charge $Q_{in}(r) = Ze \cdot \nu(r)$ in the region $r \geq r_s$, where $\nu(r)$ is given by (3.14) and has the sense which is described above. Let us remind that in this region the charge Ze of the probe particle is already completely compensated by background charge of the probe particle self-sphere. Consequently, in the region $r \geq r_s$ the charge $Q_{in}(r)$ represents a quantitative characteristic of deviation of the neutrality of the sphere with radius r, centered at the prove particle, which is exceptionally caused by the charge of $\nu(r)$ particles which are enter in this sphere from the rest of space.

From (3.12) and (3.14) it could be seen that $|Q_{in}(r)|$ in the region $r \geq r_s$ almost exponentially decreases from the maximum value $|Q_{in}(r_s)|$ down to zero, with the increasing of r. Consequently, as the requested screening length, denoted here as r_n, can be taken the root of equation

$$\frac{|Q_{in}(r)|}{|Q_{in}(r_s)|} = e^{-1}, \tag{3.15}$$

where $Q_{in}(r)$ is given by (3.12) and (3.14). In accordance with this r_n can be treated as the neutrality radius. Here, r_n is the radius of such a sphere centered at the probe particle for which charge exchange with the rest of the space becomes practically negligible in comparison with the similar exchange in the case of probe particle self-sphere, which is illustrated by Fig. 3. In the case of the initial system (see Part 1) the radius r_n can be interpreted as a radius of minimal sphere which can be considered as practically neutral one.

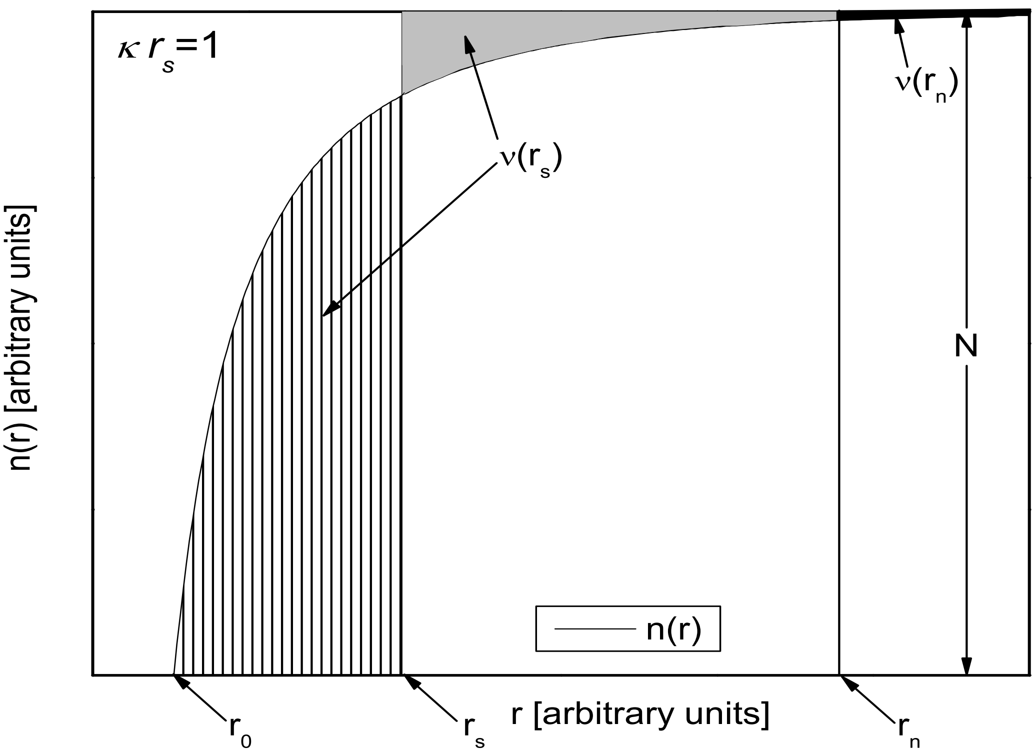

FIG. 3: The visualization of the quantity $\nu(r) = Q_{in}(r)/Ze$ for $r = r_s$ and $r = r_n$.

From (3.15) and (3.14) it follows that the characteristic length r_n represents the root of the equation:

$$\frac{1+\kappa r}{1+\kappa r_s} \cdot \exp[-\kappa(r - r_s)] = e^{-1}, \tag{3.16}$$

which can be determined only numerically. Here, it is presented in two equivalent form, namely

$$r_n = r_s \cdot \eta_s, \qquad r_n = r_\kappa \cdot \eta_\kappa, \tag{3.17}$$

where the coefficients η_s and η_κ are taken in the form

$$\eta_s = [1 + x + g(x)]/x, \qquad \eta_\kappa = 1 + x + g(x), \tag{3.18}$$

since the member $g(x)$ can be very well approximated by means of the easy expressions

$$g(x) = \begin{cases} 1.14619 - x + 0.43688 \cdot x^2 & 0 \le x \le 1, \\ 1/x - 0.5/x^2 + 0.08307/x^3 & 1 \le x < \infty, \end{cases} \tag{3.19}$$

where at the point $x = 1$ both expressions give the same value $g(1) = 0.58307$. For the coefficients η_s and η_κ we obtain then following approximate expressions

$$\eta_s = \begin{cases} \dfrac{2.14619}{x} + 0.43688 \cdot x, & 0 < x \le 1, \\ 1 + \dfrac{1}{x}\left(1 + \dfrac{1}{x} - \dfrac{0.5}{x^2} + \dfrac{0.08307}{x^3}\right), & 1 \le x < \infty, \end{cases} \tag{3.20}$$

$$\eta_\kappa = \begin{cases} 2.14619 + 0.43688 \cdot x^2, & 0 < x \le 1, \\ x + 1 + \dfrac{1}{x}\left(1 - \dfrac{0.5}{x} + \dfrac{0.08307}{x^2}\right), & 1 \le x < \infty, \end{cases} \tag{3.21}$$

where $x = \kappa r_s$. The behavior of η_s and η_κ obtained numerically from basic equation (3.16) and by approximative expressions (3.18) and (3.19), is illustrated in Fig. 4. This figure shows that the mentioned approximative expressions give very good results. From (3.20) it follows that: $\lim\limits_{x\to 0} \eta_s = \infty$ and $\lim\limits_{x\to\infty} \eta_s = 1$. Consequently, on the base of (3.17) we have it that: $r_s < r_n < \infty$ for any $\kappa r_s > 0$, and Wigner-Seitz's radius $r_s = \lim_{x\to\infty} r_n$. Then, from (3.21) it follows that: $\eta_\kappa > 1$ for $0 < \kappa r_s < \infty$. *On the base of (3.17) we have it that: $r_n > r_c$ for any $\kappa r_s > 0$.*

IV. THE SCREENING PARAMETERS OF THE TWO-COMPONENT SYSTEMS

A. "Small" characteristic lengths $r_{0;i,e}$ and non-ideality parameters $\gamma_{s;i,e}$ and $\gamma_{\kappa;i,e}$

The connection of $r_{0;i,e}$ with Landau radii $r_{L;i,e}$. In Part 2 it was shown that the simplifications which give the possibility to describe a two-component plasma in DH

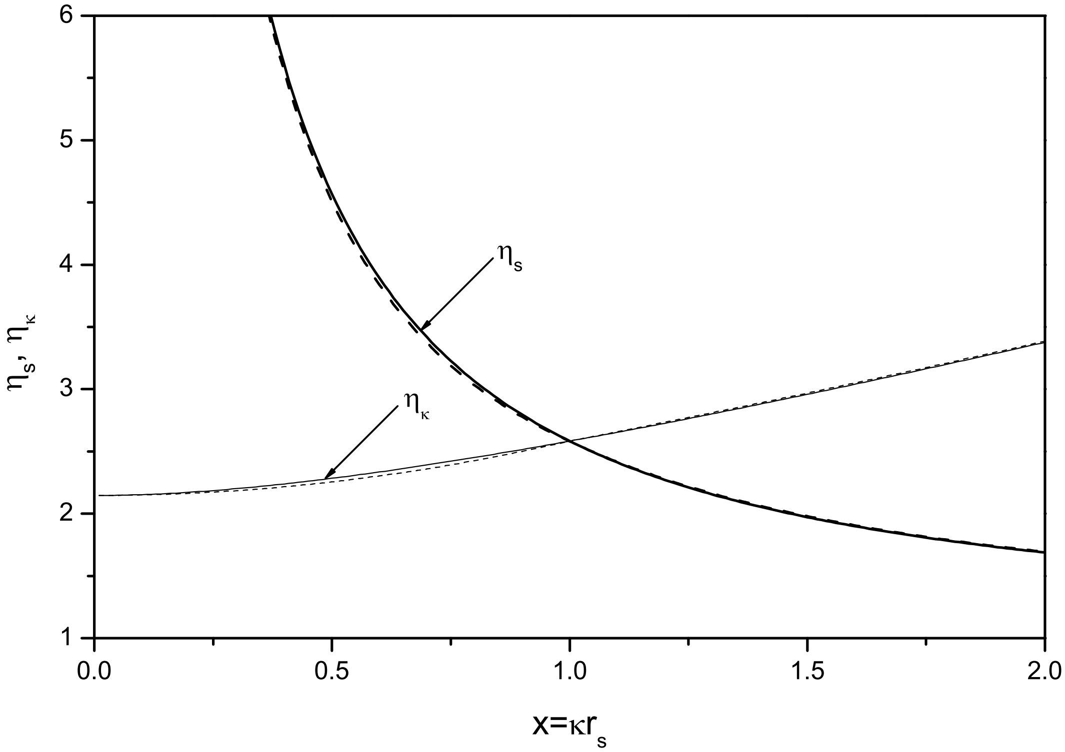

FIG. 4: The behavior of the coefficients $\eta_s = r_n/r_s$, $\eta_\kappa = r_n/r_\kappa$. Full lines show η_s and η_κ numerically determined from equation (3.16); dashed lines - the same quantities determined by approximate expressions (3.20) and (3.21).

method by means of unique screening constant are unacceptable and that its ion and electron components have to be described by own screening constant κ_i and κ_e given by Eqs. (2.8) and (2.9). One can see that these screening constants are substantially different from the DH screening constant (see Part 2). In the general case $\kappa_i \neq \kappa_e$. Unique exception is the case of completely classical plasma with $Z_i = 1$, and $T_i = T_e$, but even in this case the common value of ion and electron screening constants is significantly different from DH screening constant.

The existence of two special screening constants κ_i and κ_e causes that in the case of two-component plasma we have two groups of screening parameters, analogous to the screening parameters described in the previous Section, but for ion and electron components separately

(see Part 2). As first, we will consider "small" characteristic lengths $r_{0;i}$ and $r_{0;e}$, which are analogous to the characteristic length r_0 in the single-component case, and have the sense of radii of the spheres centered on the probe particles which are classically forbidden for the ions in the case (i) and for electrons in the case (e).

Keeping in mind Eqs. (2.5) and (2.12), as well as the facts that $(1-\alpha) \cong 1$ in the case of weakly non-ideal plasma and that in the classical case $\partial\mu_{i,e}/\partial N_{i,e} = kT_{i,e}/N_{i,e}$, we have the relation

$$\lim_{x_{i,e}\to 0} \frac{r_{0;i,e}}{r_{L;i,e}} = 1, \tag{4.1}$$

where $x_{i,e} = \kappa_{i,e} r_{s;i,e}$, and $r_{L;i}$ and $r_{L;e}$ are known ion and electron Landau's radii, namely

$$r_{L;i} = \frac{(Z_i e)^2}{kT_i}, \qquad r_{L;e} = \frac{e^2}{kT_e}. \tag{4.2}$$

One can see that $r_{L;i}$ and $r_{L;e}$ represent the approximation of $r_{0;i}$ and $r_{0;e}$ in the classical case in the regions $x_{i,e} \ll 1$. Consequently, in the case of two-component classical plasma the characteristic lengths $r_{0;i}$ and $r_{0;e}$ can be treated as the corresponding generalization of Landau's radii $r_{L;i}$ and $r_{L;e}$. It is important that, contrary to $r_{L;i,e}$ which are principally unlimited, the radii $r_{0;i,e} < r_{s;i,e}$ for any $\kappa_{i,e} r_{s;i,e} > 0$, and that Wigner-Seitz's radii $r_{s;i,e} = \lim_{x_{i,e}\to\infty} r_{0;i,e}$.

The connection of $\gamma_{s;i,e}$ and $\gamma_{\kappa;i,e}$ with non-ideality parameters $\Gamma_{i,e}$ and $\gamma_{i,e}$. On the base of Eqs. (2.5), (2.12), (4.1) and (4.2) in the case of classical plasma, we can obtain the relations

$$\lim_{x_{i,e}\to 0} \frac{\gamma_{s;i,e}}{\Gamma_{i,e}} = 1, \qquad \lim_{x_{i,e}\to 0} \frac{\gamma_{\kappa;i,e}}{\gamma_{i,e}} = 1, \tag{4.3}$$

where $x_{i,e} = \kappa_{i,e} r_{s;i,e}$, and the quantities $\Gamma_{i,e}$ and $\gamma_{i,e}$ are often used the classical ion and electron non-ideality parameters, given by relations

$$\Gamma_{i,e} = \frac{(Z_{i,e} e)^2}{kT_{i,e} \cdot r_{s;i,e}}, \qquad \gamma_{i,e} = \frac{(Z_{i,e} e)^2}{kT_{i,e} \cdot r_{\kappa;i,e}}. \tag{4.4}$$

Let us emphasize that the parameters $\Gamma_{i,e}$ and $\gamma_{i,e}$, similarly to Γ and γ in the single-component case, are also introduced from "some physical reasons" (see for example [3, 4]. However, one can see that $\Gamma_{i,e}$ and $\gamma_{i,e}$ represent the approximations of the ion and electron non-ideality parameters $\gamma_{s;i,e}$ and $\gamma_{\kappa;i,e}$ in the region $x_{i,e} \ll 1$. Consequently, $\gamma_{s;i,e}$ and $\gamma_{\kappa;i,e}$ can be treated as the generalization of ion and electron non-ideality parameters of

two-component classical plasma. The behavior of $\gamma_{s;i,e}$ and $\gamma_{\kappa;i,e}$ as functions of $\kappa_{i,e} r_{s;i,e}$ is similar to the behavior of analogous parameters in the case of single-component system (see Fig. 1).

Let us draw attention that, *contrary to the single-component case, the coefficients* $\gamma_{s;i,e}$ *and* $\gamma_{\kappa;i,e}$ *illustrate the physical inadequateness of Debye's parameters* r_D *and* n_D *for two-component systems.* Namely, in accordance with Eqs. (2.12) and (2.13) these coefficients represent the functions of the parameters $r_{s;i,e}/r_{\kappa;i,e}$. However, in two-component case we have that $r_{\kappa;i,e} \neq r_D$ and the quantity $(r_{s;i,e}/r_{\kappa;i,e})^{-3} \neq n_D$.

B. "Medium" characteristic lengths $r_{c;i,e}$

The charges $Q^{(i,e)}_{in,out}(r)$. Similarly to the single-component case, we can take the electroneutrality condition (2.14) in the form: $Q^{(i,e)}_{in}(r) + Q^{(i,e)}_{out}(r) = 0$, where $0 \leq r < \infty$, and the quantities $Q^{(i,e)}_{in}(r)$ and $Q^{(i,e)}_{out}(r)$ are given by relations

$$Q^{(i,e)}_{in}(r) = Z_{i,e}e + \int\limits_0^r \rho^{(i,e)}(r') \cdot 4\pi r'^2 dr', \qquad Q^{(i,e)}_{out}(r) = \int\limits_r^\infty \rho^{(i,e)}(r') \cdot 4\pi r'^2 dr', \tag{4.5}$$

with the charge density $\rho^{(i,e)}(r)$ defined by Eq. (2.10). Similarly to the single-component case, $Q^{(i,e)}_{in}(r)$ is the total charge of the whole sphere with radius r, centered at the probe particle, and $Q^{(i,e)}_{out}(r)$ is the total charge of the rest of space.

In all further considerations it is needed to know the charges $Q^{(i,e)}_{in}(r)$ only in the region $r_{s;i,e} \leq r < infty$. Keeping in mind Eq. (2.11), as well as the fact that $Q^{(i,e)}_{in}(r) = -Q^{(i,e)}_{out}(r)$ under the condition (2.14), we have that

$$Q^{(i,e)}_{in}(r) = Z_{i,e}e \cdot (1-\alpha) \cdot \chi(x_{i,e})\,(1+\kappa_{i,e}r)\exp(-\kappa_{i,e}r)\,, \quad r_{s;i,e} \leq r < \infty, \tag{4.6}$$

where $x_{i,e} = \kappa_{i,e} r_{s;i,e}$, $Z_e = -1$ and $\chi(x)$ is the same function as in Eq. (3.14).

For the practical applications of described method it is important that the expression (2.9) for the coefficient of the electron-ion correlation α, which figure not only in Eq. (4.6), but in expressions for all relevant quantities, can be very well approximated by two simple expressions, namely

$$\alpha \cong \frac{1}{10} \cdot x_s^2, \qquad 0 < x_s \lesssim \frac{3}{2}, \tag{4.7}$$

where the right side represents the first member of the expansion of right side of Eq. (2.9) in the series with the respect to x_s, and

$$\alpha \cong \frac{1}{10} x_s^2 \cdot \left(1 + \frac{1}{15} x_s^2\right)^{-1}, \qquad 0 < x_s \lesssim 10^{1/2}. \tag{4.8}$$

The behavior of the coefficient α, given by Eqs. (2.9), (4.7) and (4.8) is illustrated by Fig. 5.

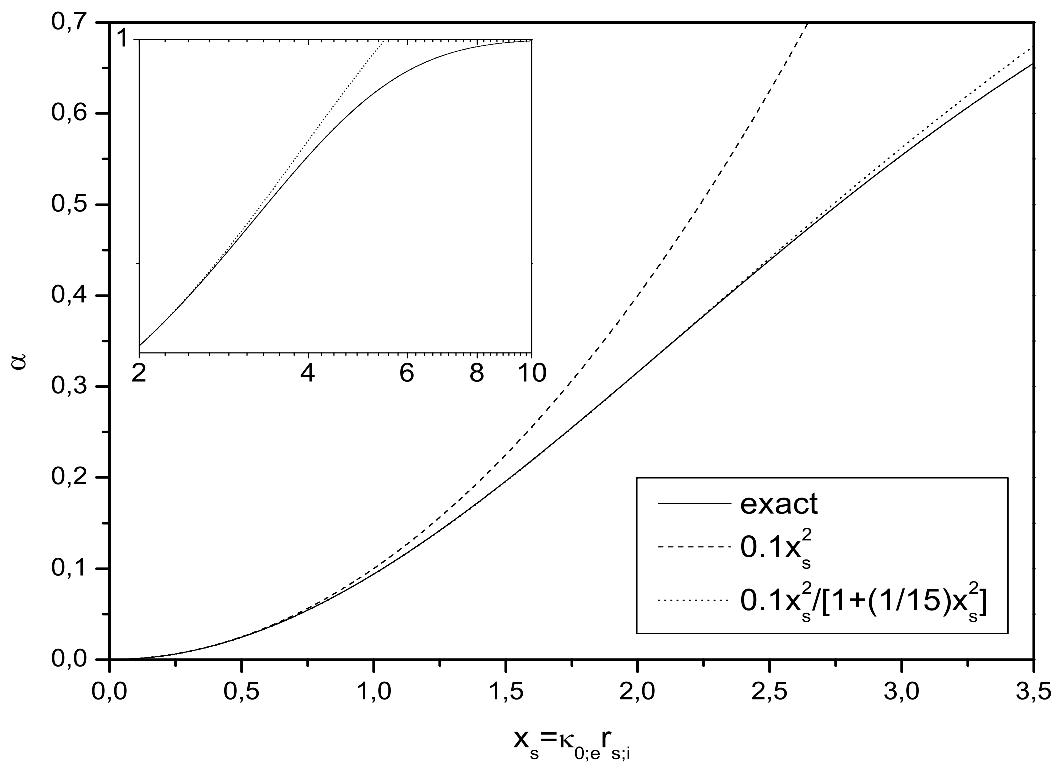

FIG. 5: The behavior of the parameter α, given by exact expression (2.9), and by approximative expressions (4.7) and (4.8).

The connection of $r_{c;i,e}$ with the radii $r_{\kappa;i,e}$. In the two-component case the ion and electron screening constants κ_i and κ_e have the sense of basic parameters of the method developed in Part 2. Consequently, the corresponding radii $r_{\kappa_i} = 1/\kappa_i$ and $r_{\kappa_e} = 1/\kappa_e$ also represent the screening parameters of this method. In order to clarify the real role of $r_{\kappa_{i,e}}$ we will introduce the characteristic lengths $r_{c;i}$ and $r_{c;e}$ which in the cases (i) and (e) have the similar sense as the radius r_c in the single-component case.

It can be shown that in the regions $0 < \kappa_{i,e} r_{s;i,e} < 1$ the parameters $r_{c;i,e}$ are the roots of equations

$$\frac{dP^{(i,e)}(r)}{dr} = 0, \tag{4.9}$$

where $P^{(i,e)}(r) = 4\pi r^2 \cdot \rho^{(i,e)}(r)$, and $\rho^{(i,e)}(r)$ is given by Eq. (2.11). Consequently, we have that in the region $0 < \kappa_{i,e} r_{s;i,e} \leq 1$ the relations

$$r_{c;i,e} = r_{\kappa;i,e}, \qquad r_{c;i,e} > r_{s;i,e} \tag{4.10}$$

are valid. Due to the behavior of $\rho^{(i,e)}(r)$, which is described in Part 2, we have it that in the case $\kappa_{i,e} r_{s;i,e} \geq 1$ the parameter $r_{c;i,e} \leq r_{s;i,e}$. In all examined cases in the region $1 < \kappa_{i,e} r_{s;i,e} < \infty$ we obtain that

$$r_{0;i,e} \leq r_{c;i,e}, \qquad r_{c;i,e} \neq r_{\kappa_{i,e}}, \tag{4.11}$$

except of the points $x_{i,e} = 7^{1/3}$, where it is $r_{c;i,e} = r_{\kappa;i,e} = r_{0;i,e}$. From just mentioned it follows that the radii r_{κ_i} and r_{κ_e} have clear physical sense only in the region $0 < \kappa_{i,e} r_{s;i,e} \leq 1$ where it is defined by the relation (4.10).

Then, similarly to the single-component case we have to examine the behavior of the charge $Q_{in}^{(i,e)}(r = r_{c;i,e})$ for $\kappa_{i,e} r_{s;i,e} \leq 1$. Since in this case $Q_{in}^{(i,e)}(r)$ is given by Eq. (4.6), we have that $Q_{in}^{(i,e)}(r_{c;i,e}) > 0.735(1-\alpha)\chi(x_{i,e}) \cdot Z_{i,e}e$. It means that $Q_{in}^{(i,e)}(r_{c;i,e})$, in accordance with the behavior of $\chi(x_{i,e})$, is comparable with the probe particle charge $Z_{i,e}e$, excluding eventually the region $x_{i,e} \gg 1$. Consequently, excluding the cases when α is close to unity, the parameters $r_{c;i}$ and $r_{c;e}$*cannot be treated as a characteristic length of full screening (neutrality) of the probe particle charges in the cases of (i) and (e).* Because of that, such characteristics lengths are determined here in the similar way as in the single-component case.

Finally, we wish to draw attention to the fact that in the weakly non-ideal plasma $(1 - \alpha \cong 1)$ the radii $r_{\kappa;i}$ and $r_{\kappa;e}$ are very closed to "medium" characteristic lengths in the corresponding single-component systems (ion gas on the negative background and electron gas on the positive background). This fact has already been considered in connection with non-applicability of DH method in the case of two-component plasmas. Namely, in [5, 6, 7, 8, 9], where the case of the classical plasma with $Z_i = 1$ and $T_i = T_e$ were considered (see Part 1), the screening constants and radius close to $\kappa_{i,e}$ and $r_{\kappa;i,e} = 1/\kappa_{i,e}$ were used,

instead of DH screening constant κ_D and radius $r_D = 1/\kappa_D$. The results obtained in this work justify such a choice.

C. "Large" characteristic lengths $r_{n;i}$ and $r_{n;e}$ as the neutrality radii

The expressions for $r_{n;i}$ and $r_{n;e}$. Similarly to Part 1, we will introduce the screening lengths $r_{n;i}$ and $r_{n;e}$ which represent the roots of the equations

$$\frac{Q_{in}^{(i,e)}(r)}{Q_{in}^{(i,e)}(r_{s;i,e})} = e^{-1}, \tag{4.12}$$

where the charge $Q_{in}^{(i,e)}(r)$ is given by Eq. (4.6) and one of Eqs. (2.9), (4.7) and (4.8). From it follows that Eq. (4.12) can be taken in the form

$$\frac{1 + \kappa_{i,e} r}{1 + \kappa_{i,e} r_{s;i,e}} \cdot \exp\left[-\kappa_{i,e}(r - r_{s;i,e})\right] = e^{-1}, \tag{4.13}$$

which is the same as the form of the corresponding equation for the radius r_n from previous Section. Consequently, we have that $r_{n;i}$ and $r_{n;e}$ are given by relations

$$r_{n;i} = r_{s;i} \cdot \eta_s(x_i), \qquad r_{n;e} = r_{s;e} \cdot \eta_s(x_e), \tag{4.14}$$

$$r_{n;i} = r_{\kappa;i} \cdot \eta_\kappa(x_i), \qquad r_{n;e} = r_{\kappa;e} \cdot \eta_\kappa(x_e), \tag{4.15}$$

where the coefficients $\eta_s(x)$ and $\eta_\kappa(x)$ are given by Eqs. (3.18)-(3.21). On the base of these expressions we have it that the relations $r_{n;i,e} > r_{s;i,e}$ and $r_{s;i,e} = \lim_{x_{i,e} to \infty} r_{n;i,e}$, as well as $r_{n;i,e} > r_{c;i,e}$, are valid in the whole regions $0 < \kappa_{i,e} r_{s;i,e} < \infty$.

The behavior of the coefficients $\eta_s(x_{i,e})$ and $\eta_\kappa(x_{i,e})$ in a wide region of $x_{i,e} = \kappa_{i,e} r_{s;i,e}$, which are given by Eqs. (2.8), (2.9), (2.13), (3.18)-(3.21), (4.14) and (4.15), is displayed in Figs. 6 and 7. These figures clearly illustrate the changes of the behavior of $\eta_s(x_{i,e})$ and $\eta_\kappa(x_{i,e})$ with the increasing of deviation of the screening constant $\kappa_{i,e}$ from their classical values in the case $x_i = x_e$. Namely, Fig. 6 shows the behavior of $\eta_s(x_i)$ and $\eta_\kappa(x_i)$ in the region $0 \le x_s = \kappa_{0,e} r_{s;e} \le 2$ where the classical and general expressions give practically the same results, while Fig. 7 is related to the region $2 < x_s \le 15$ where the classical and general expressions give very different results.

The applications and comparison with the existing experimental data. As we have already mentioned, the necessity of interpretation of experimental data caused several

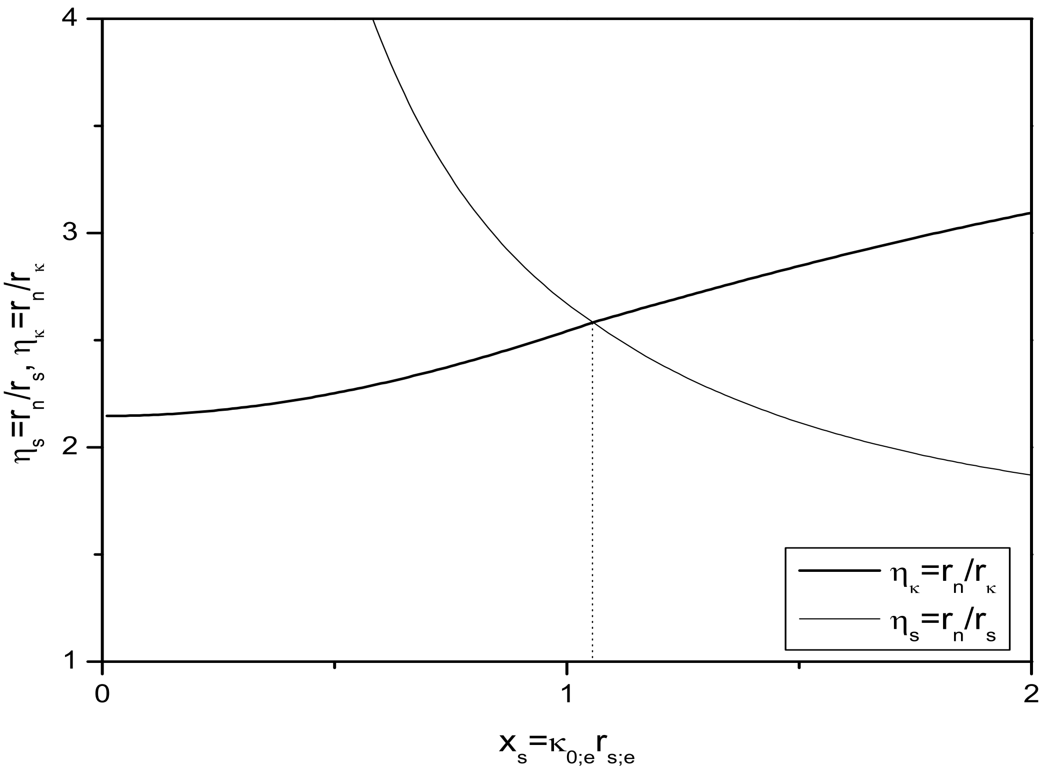

FIG. 6: The behavior of the coefficients η_s and η_κ in the region $0 \leq x_s = \kappa_{0;e}r_{s;e} \leq 2$ where the classical and general expressions for these coefficients give practically the same results. The case of plasma with $Z_i = 1$ and $T_i = T_e$, when $\kappa_{0;e}r_{s;e} = \kappa_{0;e}r_{s;i} \equiv x_s$ is presented. The shift of the crossing point of the curves presented related to the point $\kappa_{0;e}r_{s;e} = 1$ is caused by the application of the general expressions for η_s and η_κ.

attempts (see for example [10, 11, 12]) to determine of the characteristic screening length r_{scr} for two-component plasma which was taken as: $r_{scr} = k_{c;D} \cdot r_D$, where r_D is Debye's radius for two-component system (see Part 2). In order to compare the values of r_{scr} and r_n we take here r_{scr} in the form

$$r_{scr} = k_c \cdot r_{\kappa;i}, \tag{4.16}$$

where the correction factor $k_c = k_{c;D}r_D/r_{\kappa;i}$. We will have in mind that the classical plasmas with $Z_i = 1$ and $T_i = T_e$ has been considered in the above mentioned papers, and that in such plasmas: $r_{s;i} = r_{s;e}$, $\kappa_i = \kappa_e$, $r_{\kappa;i} = r_{\kappa;e}$, $r_{n;i} = r_{n;e}$ and $\eta_{\kappa;i} = \eta_{\kappa;e}$. The comparison

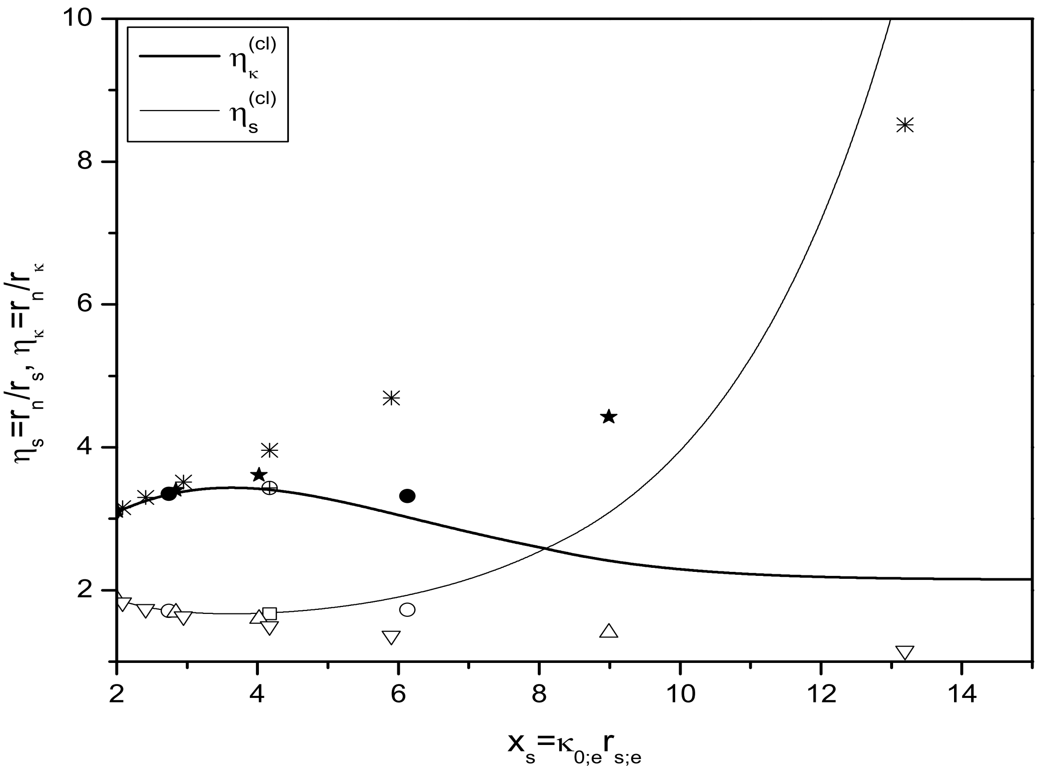

FIG. 7: The behavior of coefficients η_s and η_κ in the region $x_s = \kappa_{0;e}r_{s;e} > 2$. The curves $\eta_s^{(cl)}$ and $\eta_\kappa^{(cl)}$ show the behavior of these coefficients obtained in the classical case. The real behavior of the same coefficients obtained with the general expression (2.14) for $\kappa_{0;e}$ is presented with: $\square$- for $N_e = 10^{19}cm^{-3}$, $\circ$- for $N_e = 10^{20}cm^{-3}$, $\triangle$- for $N_e = 10^{21}cm^{-3}$, and $\bigtriangledown$- for $N_e = 10^{22}cm^{-3}$ in the case η_s, and with: $\oplus$- for $N_e = 10^{19}cm^{-3}$, $\bullet$- for $N_e = 10^{20}cm^{-3}$, $\star$- for $N_e = 10^{21}cm^{-3}$, and $*$- for $N_e = 10^{22}cm^{-3}$ in the case η_κ.

of our results with the existing experimental data is performed for several cases and is presented in Fig. 8. This figure shows the behavior of the parameter $\eta_{\kappa;i}$ determined by (4.14)-(4.15) and the correction coefficients k_c taken from [10, 11, 13, 14, 15, 16] in the region $0.6 \leq \kappa r_s \leq 1.0$. One can see a good agreement with the correction coefficients obtained from several measurements of conductivity from [14, 16].

In order to examine usage of $r_{n;i,e}$ as a screening radius in the expressions of Spitzer's type for the conductivity of plasma, we performed a calculation of conductivity for a fully

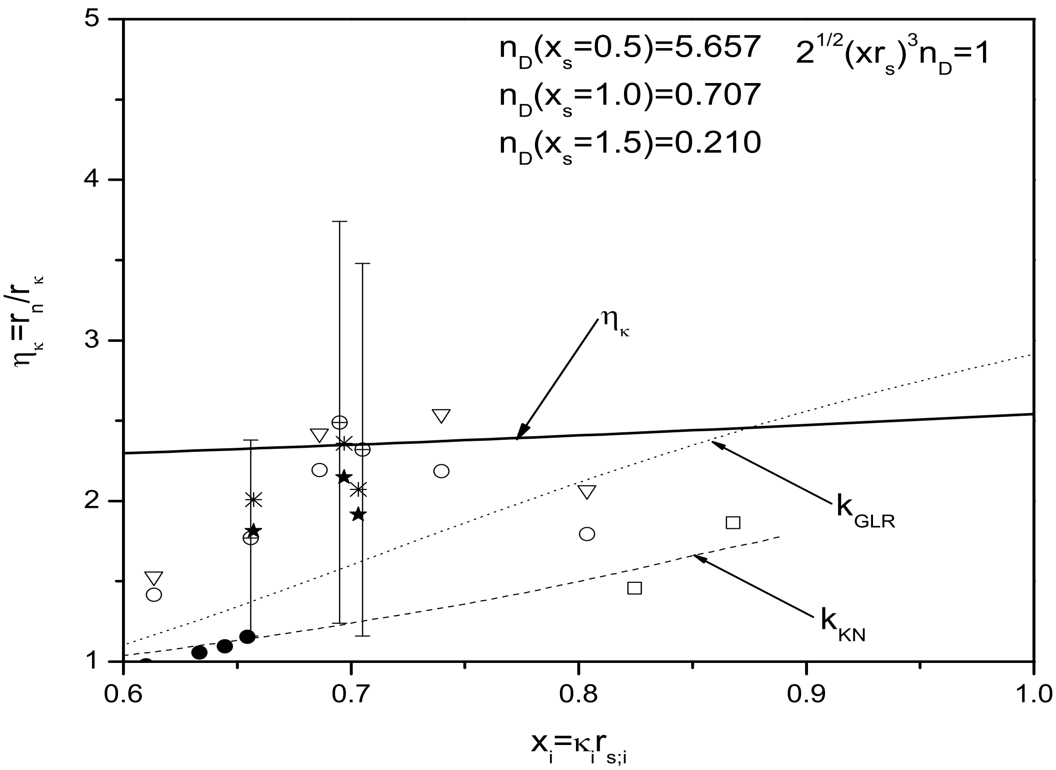

FIG. 8: The comparison of the coefficient η_κ with the correction coefficients $k_c = r_{scr}/r_{\kappa;i}$, where r_{scr} is the effective screening length determined in several papers on the base of experimental data. The cases of completely classical plasmas with $Z_i = 1$ and $T_i = T_e$ are presented. The values of k_c are shown with: $\star$ and $*$- [11, 14], $\bigtriangledown$ and $\circ$- [11, 16], $\square$- [11, 13], $\bullet$- [11, 15]. With $\oplus$ are shown the values of k_c for the same N_e and T_e as in [14], but determined by means of expression for the plasma conductivity from [17]. The curves $k_{c;KN}$ and $k_{c;GLR}$ show the behavior of k_c determined by means of analytical expressions from [10] and [11, 16], respectively.

ionized plasma with $Z_i = 1$ and $T_i = T_e = T$, where $r_{n;i} = r_{n;e}$. The cases of plasmas with $N_e = 10^{18} cm^{-3}$ and $10^{19} cm^{-3}$ in the region $10^4 K \leq T \leq 5 \cdot 10^4 K$ were considered. The conductivities were determined by means of corrected Spitzer's expression [14] where Debye's radius r_D for two-component plasma is replaced (in so called Coulomb logarithm) by screening radius r_{scr}. The calculation was performed for $r_{scr} = r_D$, $r_{scr} = k_{KN} \cdot r_D$, where k_{KN} is the corrected factor from [10], and $r_{scr} = r_{n;i}$. The results are presented in

Figs. 9 and 10. In the same figures the corresponding values of conductivity determined by improved RPA method which is applicable for dense non-ideal plasmas [8, 9, 12] are also presented.

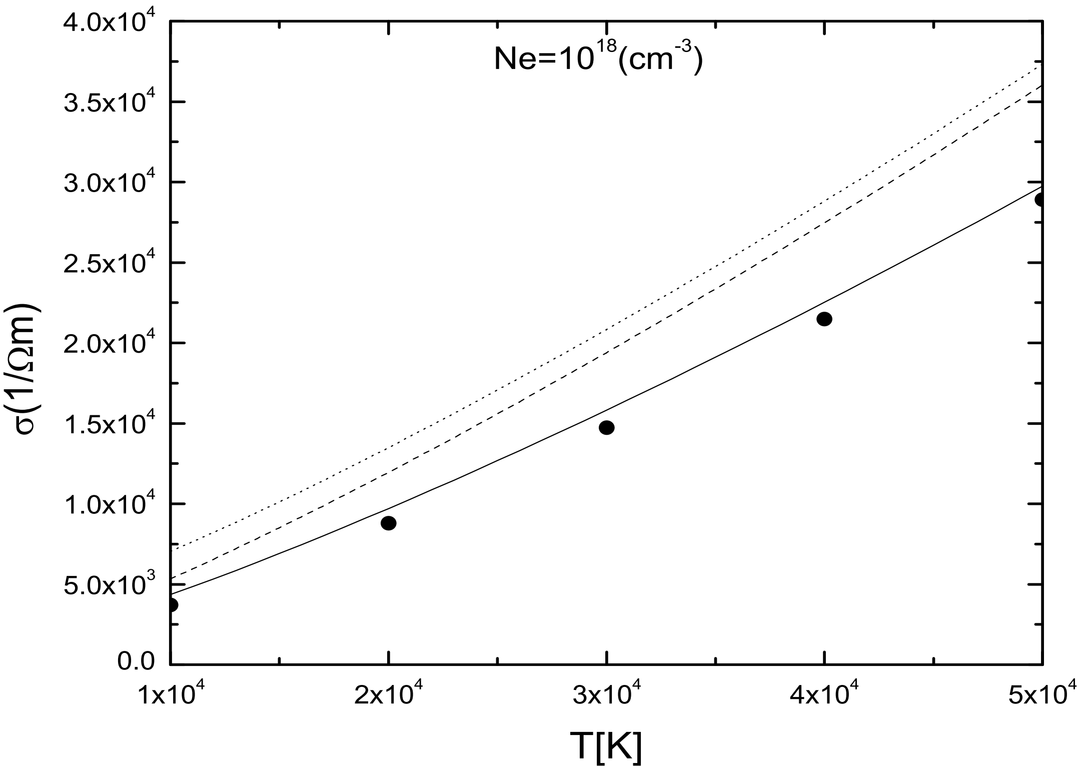

FIG. 9: The Spitzer's and RPA conductivity of plasma with $Z_i = 1$, $T_i = T_e = T$ and $N_e = 10^{18}(cm^{-3})$. The values of RPA conductivity (•) are taken form [12]. Spitzer's conductivities are determined by means of expression from [11, 14], with the corrected screening radius r_{scr}, and the corresponding calculations are performed for: $r_{scr} = r_D$ - dotted curve; $r_{scr} = r_D \cdot k_{KN}$ - dashed curve; $r_{scr} = r_{n;i}$ - full curve. Here r_D is Debye's radius for two-component plasma (see Part 2), and k_{KN} is the corrected factor from [10].

The Figs. 9 and 10 show that Spitzer's conductivity with $r_{scr} = r_{n;i}$ apparently converge to RPA conductivity with the increasing of electron density N_e from $10^{18}cm^{-3}$ to $10^{19}cm^{-3}$, while Spitzer's conductivity with $r_{scr} = r_D$ and $r_{scr} = k_{KN} \cdot r_D$ lie significantly above. One can see that use of $r_{scr} = r_{n;i}$ provides the complete agreement with the results of RPA

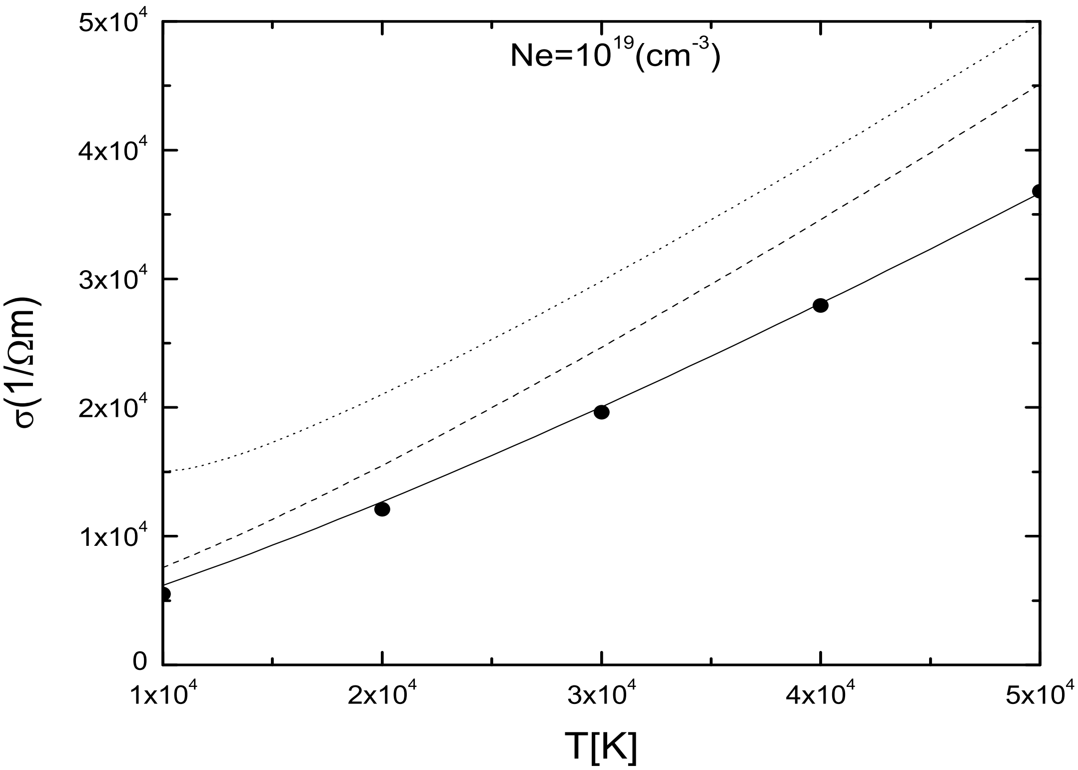

FIG. 10: Same as in Fig. 9, but for $N_e = 10^{19}(cm^{-3})$.

calculations in the case $N_e = 10^{19}cm^{-3}$. It is important that this agreement becomes better when the non-ideality degree increases.

V. CONCLUSIONS

The new model method for describing of the electrostatic screening in single- and two-components systems (electron-ion plasmas, dusty plasmas, some electrolytes, etc.) developed in Part 1 and Part 2 of this work, generates a group of new screening parameters. Here, we keep in mind "small" characteristic length r_0 and the non-ideality parameters γ_s and γ_κ in the single-component case, and the corresponding characteristic lengths $r_{0;i,e}$ and the non-ideality parameters $\gamma_{s;i,e}$ and $\gamma_{\kappa;i,e}$ in the two-component case.

In connection with the mentioned screening parameters is established that r_0 and $r_{0;i,e}$ represent the generalization of classical Landau's radii r_L and $r_{L;i,e}$, and $\gamma_{s,\kappa}$, $\gamma_{s;i,e}$ and

$\gamma_{\kappa;i,e}$ - the generalization of known classical non-ideality parameters Γ and γ in the single-component case, and $\Gamma_{i,e}$ and $\gamma_{i,e}$ in the two-component case.

Apart of that, in this paper are introduced into consideration "medium" and "large" characteristic lengths r_c and r_n in the single-component case, and $r_{c;i,e}$ and $r_{n;i,e}$ in the two-component case. The behavior of these radii is examined in the whole regions $0 < \kappa r_s < \infty$ and $0 < \kappa_{i,e} r_{s;i,e} < \infty$, where κ and $\kappa_{i,e}$ are the corresponding screening constants, and r_s and $r_{s;i,e}$- the corresponding Wigner-Seitz's radii. It was found that the considered characteristic lengths satisfy the relations

$$r_0 < r_s < r_n, \qquad r_s = \lim_{x\to\infty} r_0 = \lim_{x\to\infty} r_n, \tag{5.1}$$

$$r_{0;i,e} < r_{s;i,e} < r_{n;i,e}, \qquad r_{s;i,e} = \lim_{x_{i,e}\to\infty} r_{0;i,e} = \lim_{x_{i,e}\to\infty} r_{n;i,e}, \tag{5.2}$$

$$r_0 \le r_c < r_n, \qquad r_{0;i,e} \le r_{c;i,e} < r_{n;i,e}, \tag{5.3}$$

where $x = \kappa r_s$ and $x_{i,e} = \kappa_{i,e} r_{s;i,e}$. These relations establish the two hierarchy systems of the characteristic lengths, and causes a redefinition of Wigner-Seitz's radii as a boundary screening lengths.

Then, it was found that the radius $r_\kappa = 1/\kappa$ in the single-component case has the sense only in the region of $x \le 7^{1/3}$, where $r_\kappa = r_c$, and the radii $r_{\kappa;i,e} = 1/\kappa_{i,e}$ have the sense only in the regions $x_{i,e} \le 1$, where $r_{\kappa;i,e} = r_{c;i,e}$ in the two-component case.

The results of application of the characteristic length $r_{n;i}$ as the neutrality radius were compared in this paper with existing experimental data in the cases of the classical plasmas with $Z_i = 1$ and $T_i = T_e$. It was found their very good agreement.

Finally, we wish to draw attention that developed method is suitable for some astrophysical applications. Here we keep in mind that in outer shells of stars the physical conditions change from those which correspond to the rare, practically ideal plasma, to those which correspond to extremely dense non-ideal one. However, the method presented gives a possibility to describe the electrostatic screening of all such outer shells in the same way, by means of the obtained screening characteristics.

Acknowledgments

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APPENDIX A: THE LOWERING OF THE ATOMIC IONIZATION POTENTIAL IN PLASMA

The method developed in Part 1 and Part 2 serve for describing of the electrostatic screening in the single- and two-component systems which contain only charged particles. However, as it is well known, the presence of the neutral component in plasma can often be neglected from the aspect of inner-plasma screening. It gives the possibility to apply the results obtained in this work on the problem of the lowering of the atomic ionization potential in plasmas with the neutral component (see Part 1). In such a way we will be able to compare the results obtained within the developed and DH method. The results of this comparison will give another important example of the inapplicability of DH method.

In this context we will remind that in Part 2 the potential energies $U^{(i)}$ and $U^{(e)}$ of the ion and electron in plasma were determined. In the case $Z_i = 1$ and $T_i = T_e$, when $U^{(i)} = U^{(e)}$, the ion potential energy $U^{(i)}$ was compared with the corresponding DH ion potential energy $U_D^{(i)}$. Then, we will denote with $\triangle I_{at}$ and $\triangle I_{at;D}$ the atomic ionization potential determined within the developed and DH method, and take into account that $\triangle I_{at} \sim U^{(i)}$ and $\triangle I_{at;D} \sim U_D^{(i)}$ where the proportionality coefficients are equal or at least very closed. From here it follows the relation

$$\frac{\Delta I_{at}}{\Delta I_{at;D}} \cong \frac{U^{(i)}}{U_D^{(i)}}. \tag{A1}$$

The behavior of the right side of this relation is shown in Fig. 5 of Part 2. Accordingly to this figure we can conclude that $\Delta I_{at}/\Delta I_{at;D} < 0.8$ in the whole region $0 < \kappa_i r_{s;i} < \infty$. It is important to noted this fact, since the handbooks (see also [3]), often used in laboratories, recommend just DH lowering of the atom ionization potential $\Delta I_{at;D}$, which would not be

taken as one of screening characteristics of electron-ion plasmas.

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